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HYBRID TDOA/AOA GEOLOCATION USING OPTIMIZATION SOLUTION

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ABSTRACT

Recently, mobile location estimate problem is a significant issue in many industrial fields such as radar, wireless communication, target tracking, and various types of location based services. Time difference of arrival (TDOA) and angle of arrival (AOA) are typical geolocation techniques for estimation of mobile location. In this paper, we suggest a hybrid TDOA/AOA geolocation equation as a simple matrix form. We can derive the objective function to minimize the estimation error with using geolocation algorithm. Also, Nelder-Mead simplex method based optimization solution is proposed. We confirm the performance of our proposed algorithms with a simulation result.

KEYWORDS: TDOA, AOA, Geolocation, Optimization Problem, Nelder-Mead Simplex Method

INTRODUCTION

Among various geolocation techniques, the global positioning system (GPS) is the most widely used localization method. However, GPS has a limitation to estimate the accurate location of a mobile in downtown or indoor environment since GPS necessarily requires a satellite signal. Also, jamming is one of the reasons which decrease the localization performance. For a few years, many geolocation methods using distributed receivers such as time of arrival (TOA) and received signal strength (RSS) method have been actively researched. Recently, the location estimate systems based on TDOA and AOA method have received great attention. The TDOA method uses time difference of signal arrival from mobile to each fixed base station for estimating the mobile location. TDOA based geolocation does not require a synchronization among base stations. Due to this strength of TDOA method, it has been broadly used in real time positioning systems. AOA method uses the multiple antenna arrays in base stations to measure the arrival angle of transmitted signal from a mobile. Using the measured angle data, the geolocation equation can be formulated. In this paper, we suggest a hybrid TDOA/AOA method in order to determine the location of a mobile [1].

In both TDOA and AOA based geolocation systems, the environmental noise is a major factor which causes reduced precision in estimation. Musicki [2] suggested a mobile emitter geolocation method using Gaussian mixture presentation of measurements-integrated track splitting filter. Peng [3] proposed geolocation using a distributed AOA and orientation approach for wireless sensor networks under the assumption that all unknown sensors are capable of detecting angles of the incident signal from the neighboring nodes. To estimate the target's accurate location, Chan [4] suggested a TOA and TDOA based localization algorithm through an approximate maximum likelihood method. Chan showed the performance of the proposed algorithm compared with Cramer-Rao lower bound (CRLB).

In this paper, we propose Nelder-Mead simplex method based optimization solution for minimizing the error between the real location and the estimate of a mobile. The Nelder-Mead simplex method is a commonly used optimization method to find the minimum of an objective function in a multiple dimensional space. Several iterations using three variables of an objective function are performed until the estimation error is minimized. The organization of this paper is as follows. In Section 2, a mobile localization equation using TDOA and AOA measurements is formulated. Also the objective function which demonstrates the estimation error about a mobile location is derived. In Section 3, Nelder-Mead simplex method applies to the objective function to minimize the estimation error. Section 4 expresses the validity of the proposed algorithm through a simulation result. Finally, conclusion is presented in Section 5.

GEOLOCATION FORMULATION WITH TDOA/AOA MEASUREMENTS

Among diverse location estimate schemes, the hybrid method using TDOA and AOA measurements is the most efficient one. In this section, the location estimate formula which is composed with the TDOA and AOA data is derived for optimization problem. In order to apply Nelder-Mead simplex method to mobile geolocation formula, at least three base stations are needed in two dimensional coordinates. First, we obtain the major geolocation equations which use the TDOA and AOA measurements separately. With combining these two major formulas, hybrid TODA/AOA geolocation equation is formulated as a matrix form.

When the TDOA measurements are available at each base station, the geolocation formula of a mobile can be acquired. Let $\mathbf{ms} = [x, y]^T$ be an unknown mobile location that we try to estimate and let $\mathbf{bs}_i = [x_i, y_i]^T$ be a known location of based station. The TDOA measurements can be obtained as follows

$$r_{i1}^0 = r_i^0 - r_1^0 = \sqrt{(x-x_i)^2 + (y-y_i)^2} - \sqrt{(x-x_1)^2 + (y-y_1)^2}. \quad i = 2, 3, \cdots, M \tag{1}$$

where r_i^0 denotes the range from a mobile to the i-th base station, r_{i1}^0 is the measured range difference data between i-th receiver and the first receiver, and c means the speed of signal propagation. We can derive the following linear equation using the TDOA data.

$$(x_i - x_1)(x - x_1) + (y_i - y_1)(y - y_1) + r_{i1}^0 r_1^0 = \frac{1}{2}\left[(x_i - x_1)^2 + (y_i - y_1)^2 - \left(r_{i1}^0\right)^2\right] \tag{2}$$

The difference between the estimated location and the true location of a mobile occurs from the environmental and nonline-of-sight noise in TDOA data. The AOA data of the propagated signal from the mobile at the i-th base station, denoted by ϕ_i, can be formulated with the locations of a mobile and a base station as follows

$$\begin{aligned} \tan(\phi_i) &= \frac{\sin(\phi_i)}{\cos(\phi_i)} \\ &= \frac{y - y_i}{x - x_i}. \end{aligned} \tag{3}$$

The equation (3) can be rewritten as a following equation,

$$x\sin(\phi_i) - y\cos(\phi_i) = x_i \sin(\phi_i) - y_i \cos(\phi_i). \tag{4}$$

The measured AOA data ϕ_i at the i-th base station contains the environmental noise in real case. These differences between TDOA/AOA measurement and a true data lead to an inaccuracy in a mobile location estimation.

The combination of different types of the measurement data can improve the location estimation performance and reduce the number of base stations due to the increase of data set. With using equations (2) and (4), the hybrid TDOA and AOA based geolocation formula can be denoted as a matrix form as follows

$$\mathbf{A\theta} = \mathbf{b} \tag{5}$$

Where

$$\mathbf{A} = \begin{bmatrix} x_2 - x_1 & y_2 - y_1 & r_{21}^0 \\ \vdots & \vdots & \vdots \\ x_M - x_1 & y_M - y_1 & r_{M1}^0 \\ \sin(\phi_1) & -\cos(\phi_1) & 0 \\ \sin(\phi_2) & -\cos(\phi_2) & 0 \\ \vdots & \vdots & \vdots \\ \sin(\phi_M) & -\cos(\phi_M) & 0 \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} \frac{1}{2}\left[(x_2 - x_1)^2 + (y_2 - y_1)^2 - (r_{21}^0)^2\right] \\ \vdots \\ \frac{1}{2}\left[(x_2 - x_1)^2 + (y_2 - y_1)^2 - (r_{21}^0)^2\right] \\ 0 \\ (x_2 - x_1)\sin(\phi_2) - (y_2 - y_1)\cos(\phi_2) \\ \vdots \\ (x_M - x_1)\sin(\phi_M) - (y_M - y_1)\cos(\phi_M) \end{bmatrix}$$

The parameter vector $\mathbf{\theta} = \begin{bmatrix} x - x_1 & y - y_1 & r_1^0 \end{bmatrix}^T$ in equation (5) is the solution of formulated hybrid TDOA and AOA based geolocation equation that includes the location of a mobile. Combining TDOA and AOA formula, M base stations can obtain a location of mobile using $(2M - 1)$ measurement data set. In order to solve this TDOA/AOA geolocation problem denoted by equation (5), we propose the Nelder-Mead simplex method based optimization algorithm.

In order to acquire the optimized solution of a mobile location, the difference between the estimated location and the real location needs to be minimized. In equation (5), the set of linear equations is inconsistent due to the noisy elements in matrix $\mathbf{A}$ and vector $\mathbf{b}$. The objective function for minimizing the estimation error can be obtained as a following equation.

$$F\left(\hat{\boldsymbol{\theta}}\right)=\left(\mathbf{A}\hat{\boldsymbol{\theta}}-\mathbf{b}\right)^T\left(\mathbf{A}\hat{\boldsymbol{\theta}}-\mathbf{b}\right) \tag{6}$$

where $\hat{\boldsymbol{\theta}}=\left[\hat{x}-x_1, \hat{y}-y_1, \hat{r}_1^0\right]^T$ is an optimization variable vector. In order to solve the objective function which is denoted by equation (6), the Nelder-Mead simplex method applies to $F\left(\hat{\boldsymbol{\theta}}\right)$.

OPTIMIZED SOLUTION USING NELDER-MEAD SIMPLEX METHOD

In this section, the optimized solution of the geolocation objective function is derived for minimizing the error between the estimated location and the real location of a mobile. Nelder-Mead simplex method is an optimized iteration process that is widely used in problems with nonlinear objective functions. This method can be applied to nonlinear optimization problems for which the derivatives of an objective function are very restricted to be expressed in a closed form. Nelder-Mead simplex method consists of four basic processes such as reflection, expansion, contraction, and shrink.

We denote a simplex with the initial values of three design variables $\hat{\boldsymbol{\theta}}_1$, $\hat{\boldsymbol{\theta}}_2$, and $\hat{\boldsymbol{\theta}}_3$. They can be ordered with best, good, and worst vertices ($\hat{\boldsymbol{\theta}}_b, \hat{\boldsymbol{\theta}}_g, \hat{\boldsymbol{\theta}}_w$) according to the objective function value as

$$F\left(\hat{\boldsymbol{\theta}}_b\right)<F\left(\hat{\boldsymbol{\theta}}_g\right)<F\left(\hat{\boldsymbol{\theta}}_w\right) \tag{7}$$

These three variables are replaced with expansion, contraction, and shrink process on the basis of the value of reflection vertex. The reflected point $\hat{\boldsymbol{\theta}}_r$ can be obtained as a following equation

$$\hat{\boldsymbol{\theta}}_r=\hat{\boldsymbol{\theta}}_m+\alpha\left(\hat{\boldsymbol{\theta}}_m-\hat{\boldsymbol{\theta}}_w\right) \tag{8}$$

where $\hat{\boldsymbol{\theta}}_m$ is the midpoint which can be calculated as $\hat{\boldsymbol{\theta}}_m=0.5\left(\hat{\boldsymbol{\theta}}_b+\hat{\boldsymbol{\theta}}_g\right)$ and the reflection parameter α is a positive real number. If $f\left(\hat{\boldsymbol{\theta}}_b\right)<f\left(\hat{\boldsymbol{\theta}}_r\right)<f\left(\hat{\boldsymbol{\theta}}_g\right)$, replace $\hat{\boldsymbol{\theta}}_w$ with $\hat{\boldsymbol{\theta}}_r$. $\hat{\boldsymbol{\theta}}_b$, $\hat{\boldsymbol{\theta}}_r$, and $\hat{\boldsymbol{\theta}}_g$ can be the new design variables of next iteration.

If $f(\hat{\boldsymbol{\theta}}_r) < f(\hat{\boldsymbol{\theta}}_b)$, the expansion process can be proceeded. The expansion point $\hat{\boldsymbol{\theta}}_e$ can be computed as follows

$$\hat{\boldsymbol{\theta}}_e = \hat{\boldsymbol{\theta}}_m + \beta\left(\hat{\boldsymbol{\theta}}_r - \hat{\boldsymbol{\theta}}_m\right) \tag{9}$$

with an expansion parameter β with $\beta > 1$. If $f(\hat{\boldsymbol{\theta}}_e) < f(\hat{\boldsymbol{\theta}}_r)$, the vertices of next iteration are $\hat{\boldsymbol{\theta}}_b$, $\hat{\boldsymbol{\theta}}_e$, and $\hat{\boldsymbol{\theta}}_g$. Otherwise, $\hat{\boldsymbol{\theta}}_w$ can be replaced with $\hat{\boldsymbol{\theta}}_r$. If $f(\hat{\boldsymbol{\theta}}_g) < f(\hat{\boldsymbol{\theta}}_r) < f(\hat{\boldsymbol{\theta}}_w)$, we can calculate the outside contraction point ($\hat{\boldsymbol{\theta}}_{oc}$) as follows

$$\hat{\boldsymbol{\theta}}_{oc} = \hat{\boldsymbol{\theta}}_m + \gamma\left(\hat{\boldsymbol{\theta}}_r - \hat{\boldsymbol{\theta}}_m\right) \tag{10}$$

with $0 < \gamma < 1$. If $f(\hat{\boldsymbol{\theta}}_{oc}) < f(\hat{\boldsymbol{\theta}}_r)$, replace $\hat{\boldsymbol{\theta}}_w$ with $\hat{\boldsymbol{\theta}}_{oc}$. If the value of $f(\hat{\boldsymbol{\theta}}_r)$ is smaller than that of $f(\hat{\boldsymbol{\theta}}_{oc})$, we can proceed to the shrink step. We can obtain the new variables of $\hat{\boldsymbol{\theta}}_g$ and $\hat{\boldsymbol{\theta}}_w$ through the shrink process as following equations.

$$\begin{aligned} \hat{\boldsymbol{\theta}}'_g &= \hat{\boldsymbol{\theta}}_b + \eta\left(\hat{\boldsymbol{\theta}}_g - \hat{\boldsymbol{\theta}}_b\right) \\ \hat{\boldsymbol{\theta}}'_w &= \hat{\boldsymbol{\theta}}_b + \eta\left(\hat{\boldsymbol{\theta}}_w - \hat{\boldsymbol{\theta}}_b\right) \end{aligned} \tag{11}$$

where the shrink parameter satisfies $0 < \eta < 1$. If $f(\boldsymbol{\delta}_w) < f(\boldsymbol{\delta}_r)$, compute the inside contraction point as

$$\hat{\boldsymbol{\theta}}_{ic} = \hat{\boldsymbol{\theta}}_m - \gamma\left(\hat{\boldsymbol{\theta}}_r - \hat{\boldsymbol{\theta}}_m\right) \tag{12}$$

If $f(\boldsymbol{\delta}_{ic}) < f(\boldsymbol{\delta}_w)$, the vertices of next iteration are $\hat{\boldsymbol{\theta}}_b$, $\hat{\boldsymbol{\theta}}_g$, and $\hat{\boldsymbol{\theta}}_{ic}$. Otherwise, we can obtain the new variable set through the shrink procedure.

SIMULATION RESULT

In this section, the performance of a proposed hybrid TDOA/AOA location estimate algorithm using Nelder-Mead simplex method is confirmed by some simulations. In our simulations, the location of a mobile is estimated by using TDOA and AOA data from five receivers. The locations of each receiver are (0, 0), (0, 100), (50, 0), (50, 100), and (100, 50) km, respectively. We suppose that a mobile moves at a constant speed following a zig-zag trajectory. The measured TDOA/AOA data contain some environmental and nonline-of-sight errors. These errors are assumed to follow the Gaussian distribution with a variance 0.1 and 0.01, respectively. For the calculational simplicity, the signal propagation speed c is supposed as 1km/s. We set the three initial variable $\hat{\boldsymbol{\theta}}_1$, $\hat{\boldsymbol{\theta}}_2$, and $\hat{\boldsymbol{\theta}}_3$ as (0, 0), (10, 0), and (0, 10) km, respectively. Each parameter per the seperate process, α, β, γ, and η are chosen to be 1, 2, 0.5, and 0.5.

Figure 1 describes the estimated trajectory comparison between Nelder-Mead simplex method and standard least square (LS) method. The thick solid line is the estimated trajectory using Nelder-Mead simplex method and the dotted line means the LS based trajectory estimation. As shown in Figure 1, the estimated trajectory using the proposed Nelder-Mead simplex method shows the better performance than LS based estimation.

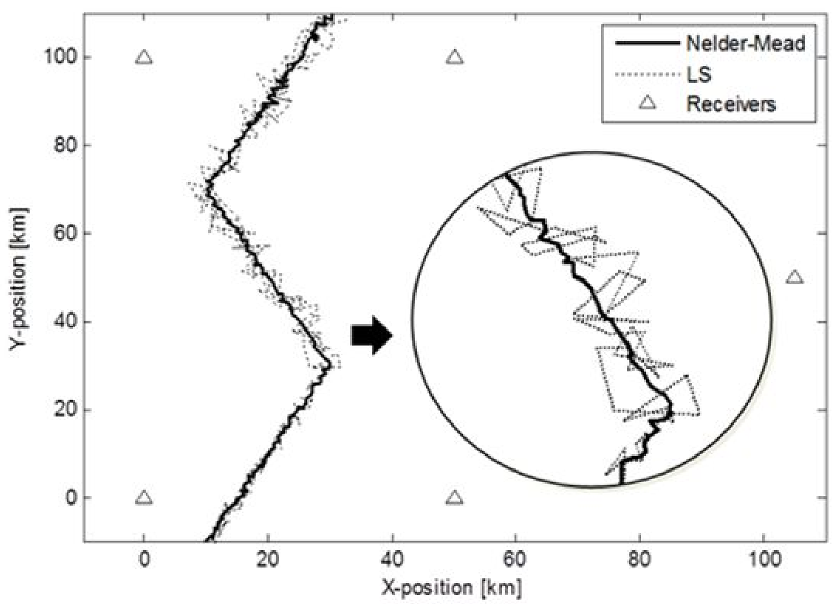

Figure 1: Estimated Trajectory Comparison between Nelder-Mead Method and LS Algorithm

CONCLUSIONS

In this paper, the hybrid TDOA/AOA geolocation method with an optimization solution was suggested through Nelder-Mead simplex method. With TDOA and AOA measurement data, the localization equation was formulated as a simple matrix form. Both environmental and NLOS noise causes the location estimate error which is a significant problem in geolocation technique. In order to mitigate the estimation error, Nelder-Mead simplex method based optimization algorithm was applied to the objective function which means the amount of error between the estimated location and the real location. As the iteration procedure was performed, the optimized vertex which minimizes the estimation error was acquired. The simulation result confirmed the performance of our proposed optimization method.

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